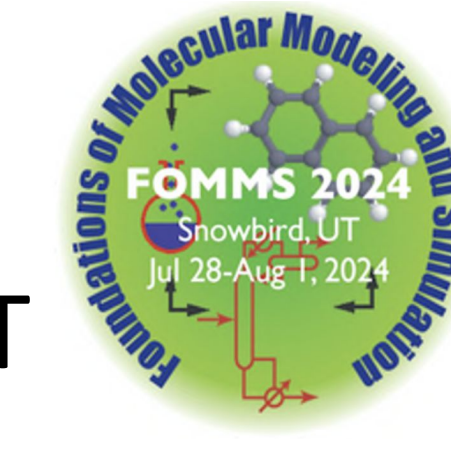


# Exploring the State-Dependence of Classical Pair Potentials with Neutron Scattering



Brennon L. Shanks, Harry W. Sullivan, Michael P. Hoepfner  
University of Utah, Department of Chemical Engineering, Salt Lake City, UT



## MOTIVATION

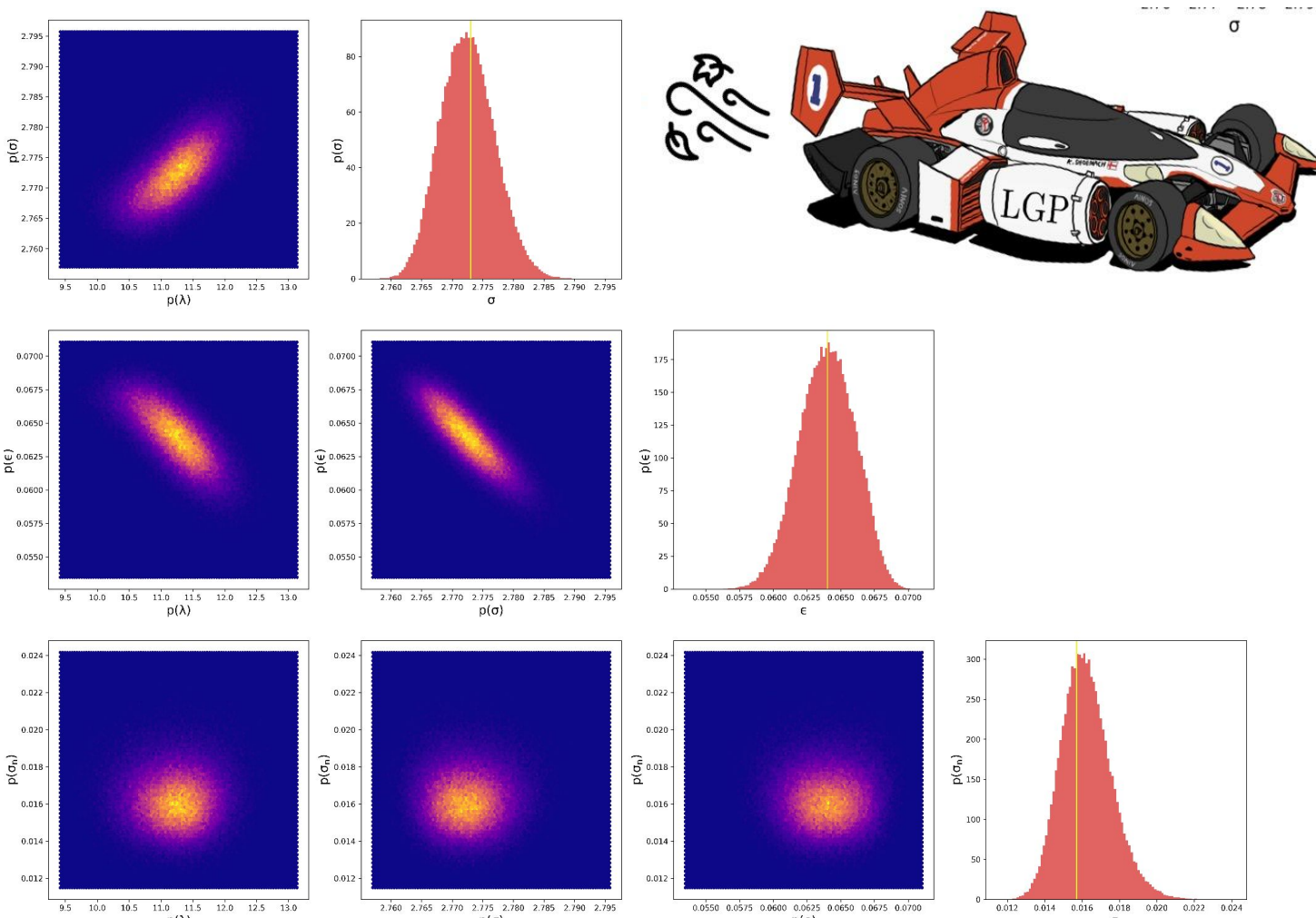
The prediction of interaction potentials from pair correlation functions, the so called inverse problem of statistical mechanics, provides a route to develop better force fields for molecular modeling.

Inverse techniques are widely used for coarse graining, but have not established a foothold in the analysis of experimental data.

**Question: Can we extend these inverse techniques to experimental scattering data to improve fluid models?**

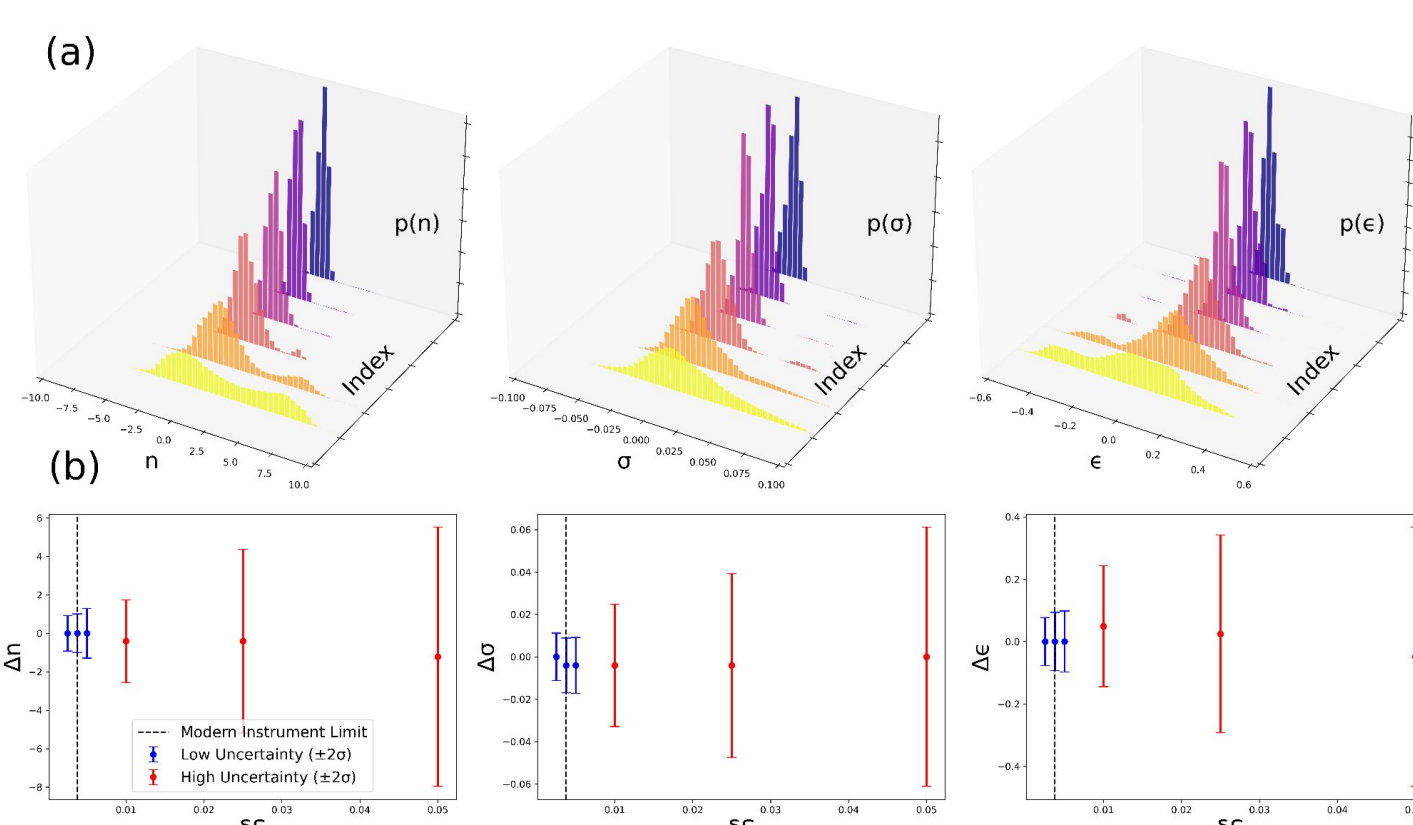
## Pair Potentials are Sensitive to Liquid Structure

We recently designed a Bayesian UQ method for force fields with local Gaussian process surrogate models over the radial distribution function [1].



Using this model, we tested how sensitive (n-6) Mie parameters are to structure factors at varying degrees of noise.

Bayesian UQ was used to study if neutron scattering data is a good FF benchmark.



We discovered that pair potentials are more sensitive to scattering data than previously believed [2].

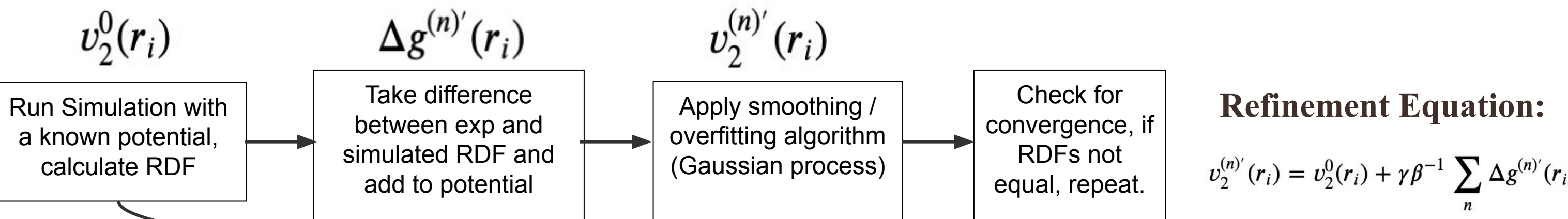
## Key Takeaways

- Neutron scattering data is a viable target to train force fields.
- We have created rigorous methods for Bayesian UQ of force fields to complex scattering data.
- **What we can learn from scattering with respect to interatomic interactions?**

## LEARNING INTERACTION POTENTIALS FROM SCATTERING DATA AND RELATIONS TO QUANTUM MECHANICS

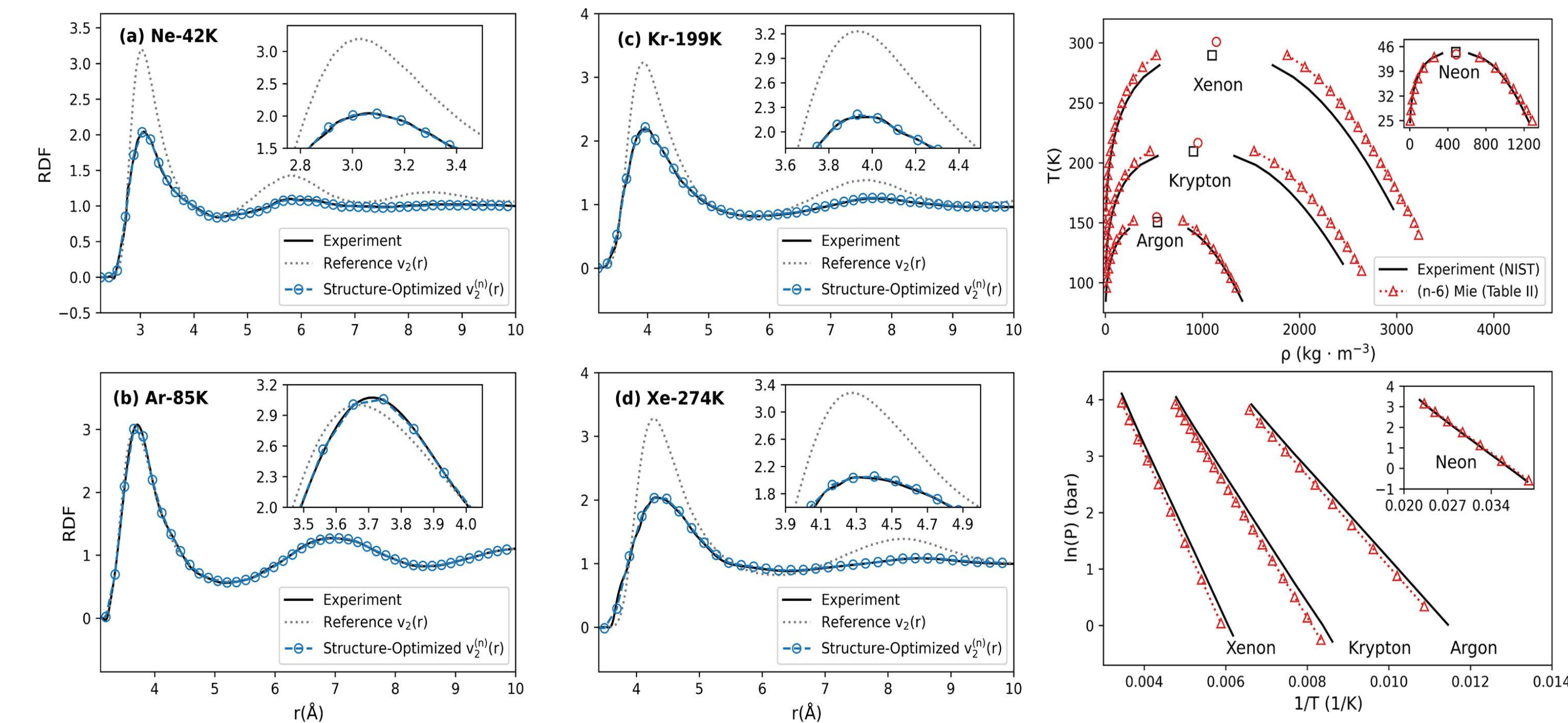
### Structure Optimized Potential Refinement (SOPR) [3]

Probabilistic iterative Boltzmann inversion for learning transferable potentials from experimental scattering data.



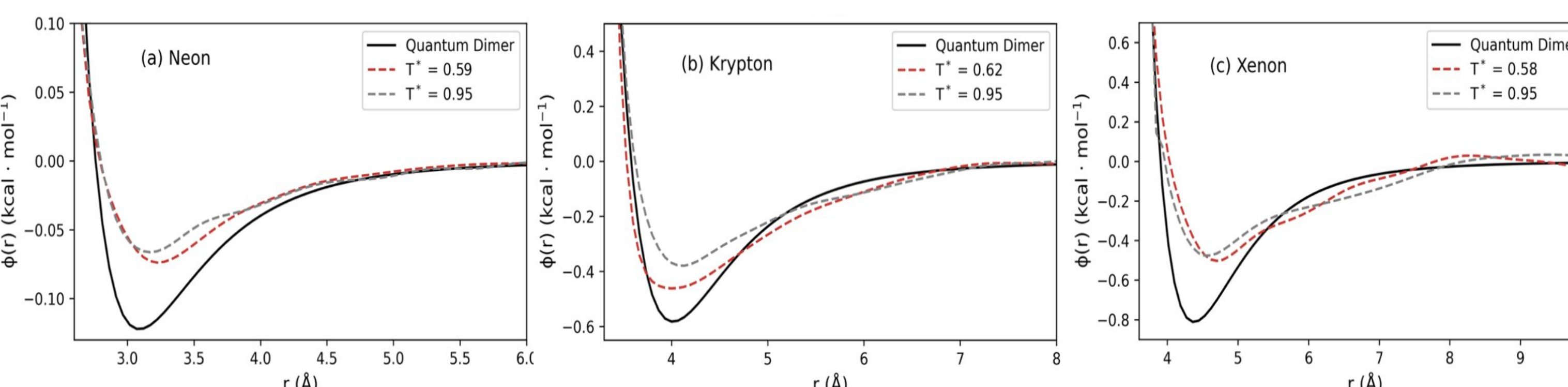
### Refinement Equation:

$$v_2^{(n)'}(r_i) = v_2^{(n)}(r_i) + \gamma \beta^{-1} \sum_n \Delta g^{(n)}(r_i)$$

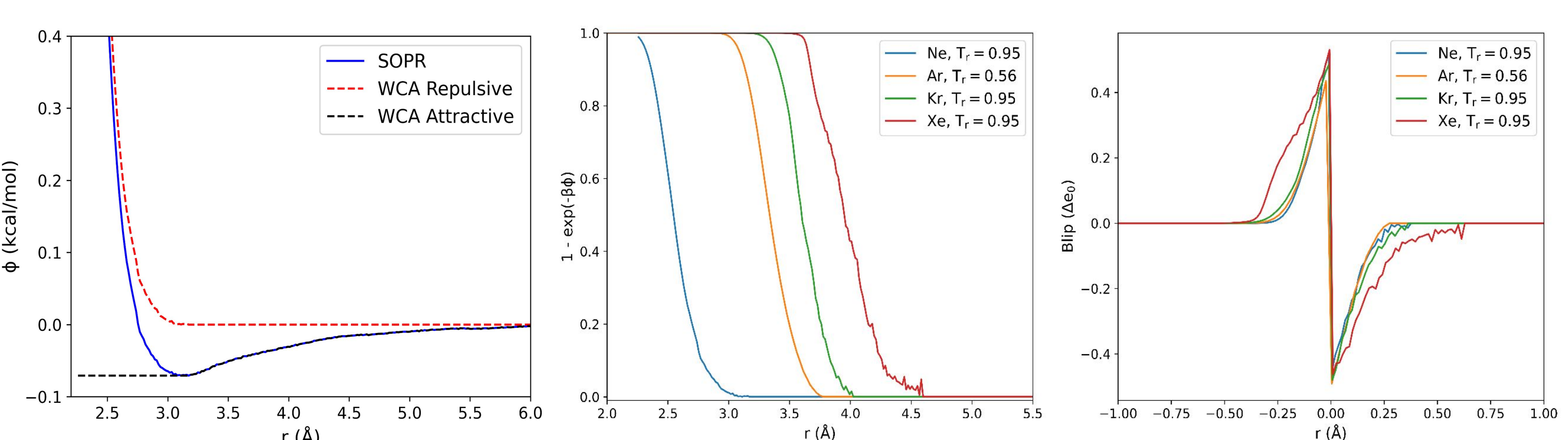


### Comparing SOPR and Quantum Pair Potentials

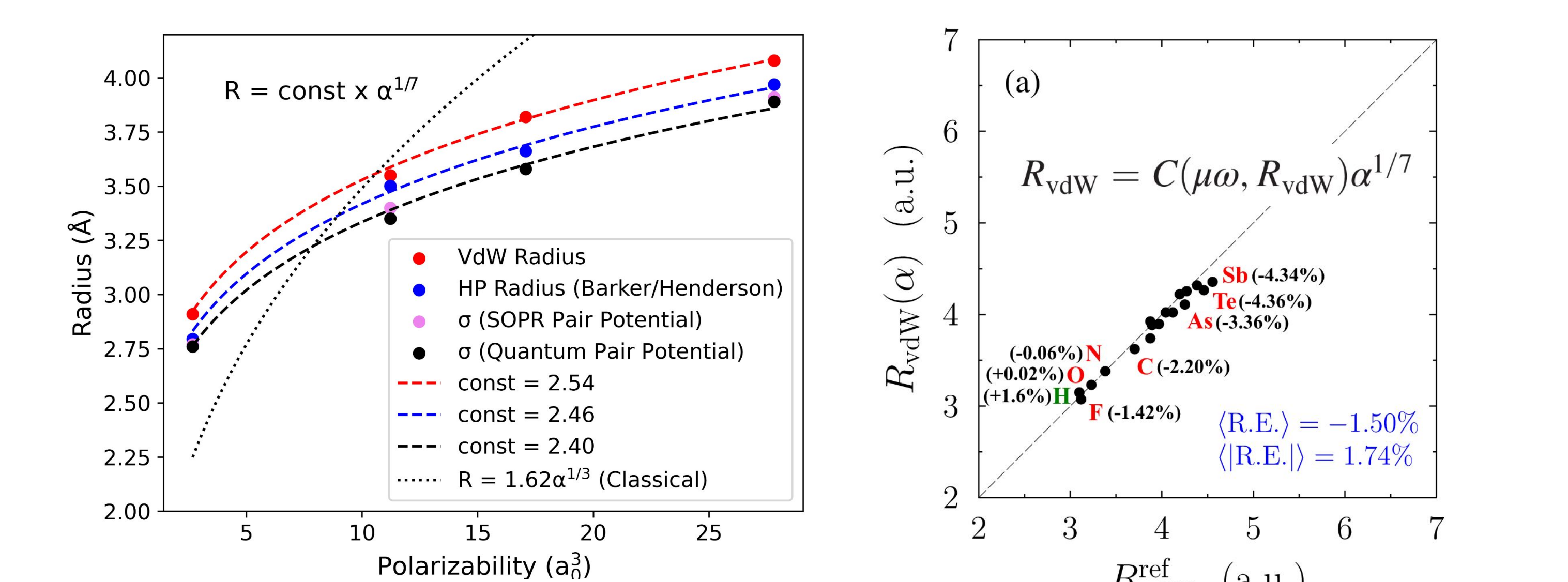
SOPR vs Quantum Potential for Ne, Kr, Xe



### Weeks-Chandler-Anderson Decomposition and Computation of Hard Particle Diameter and Blip Function

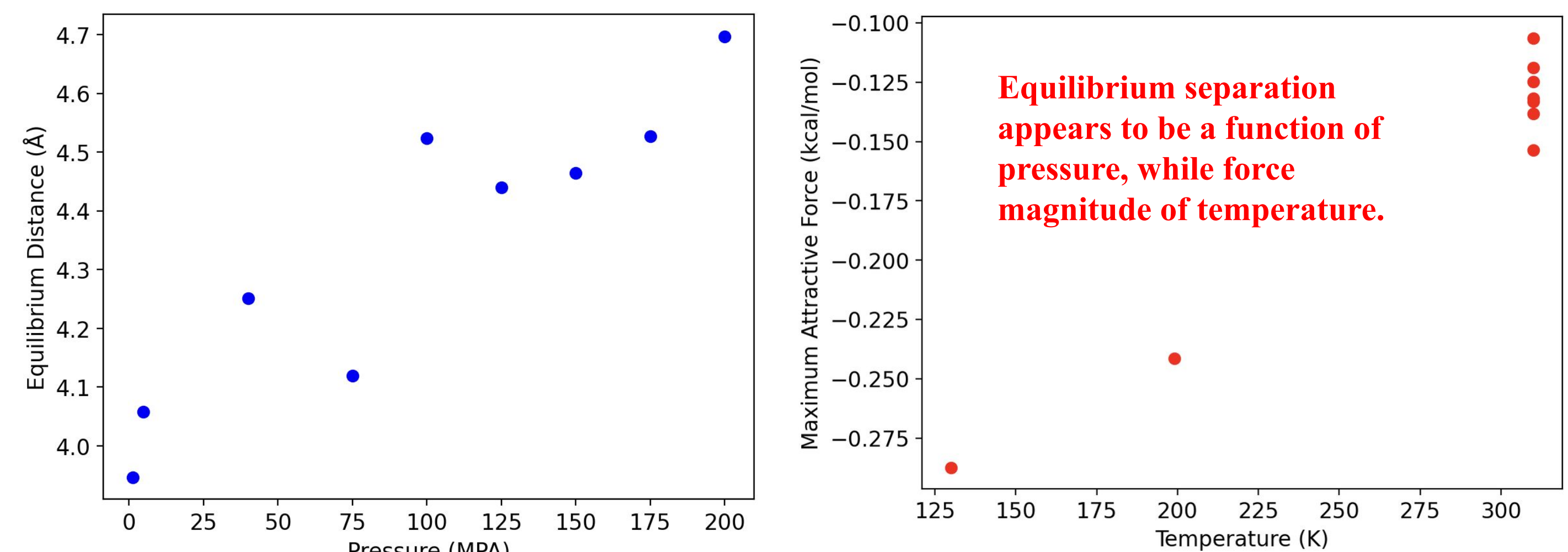
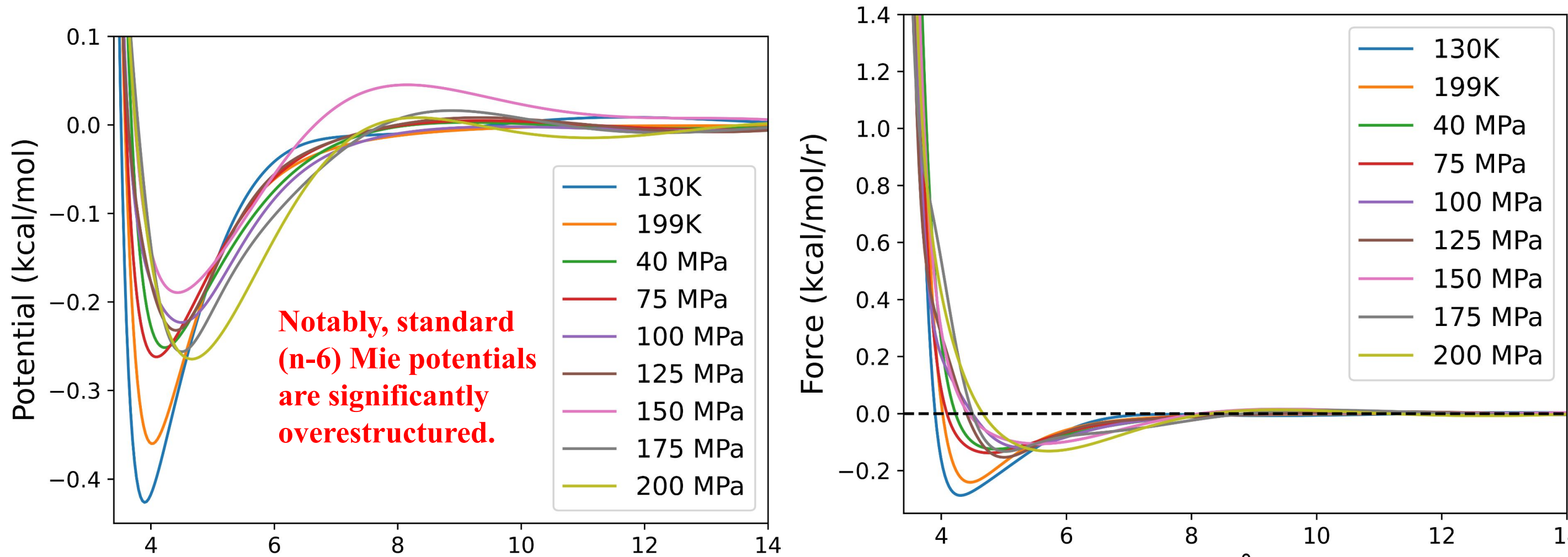
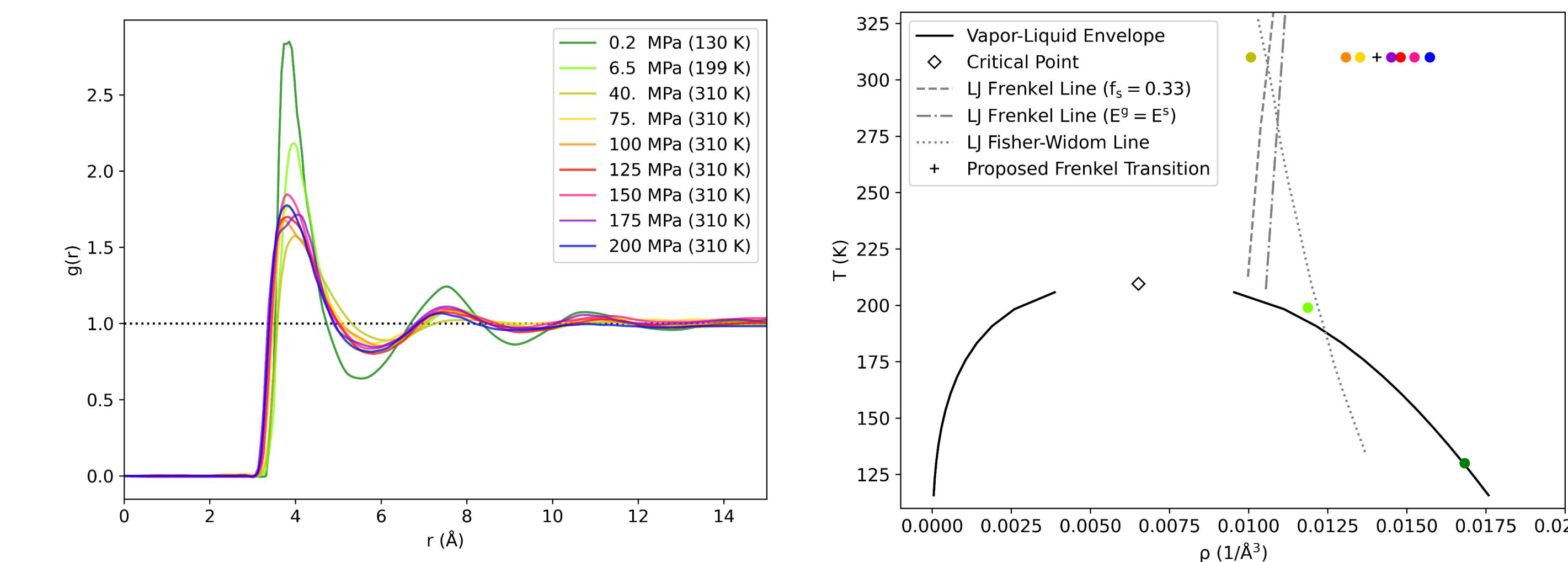


### SOPR Derived Collision Diameters Scale with Polarizability as a Quantum Drude Oscillator [4]

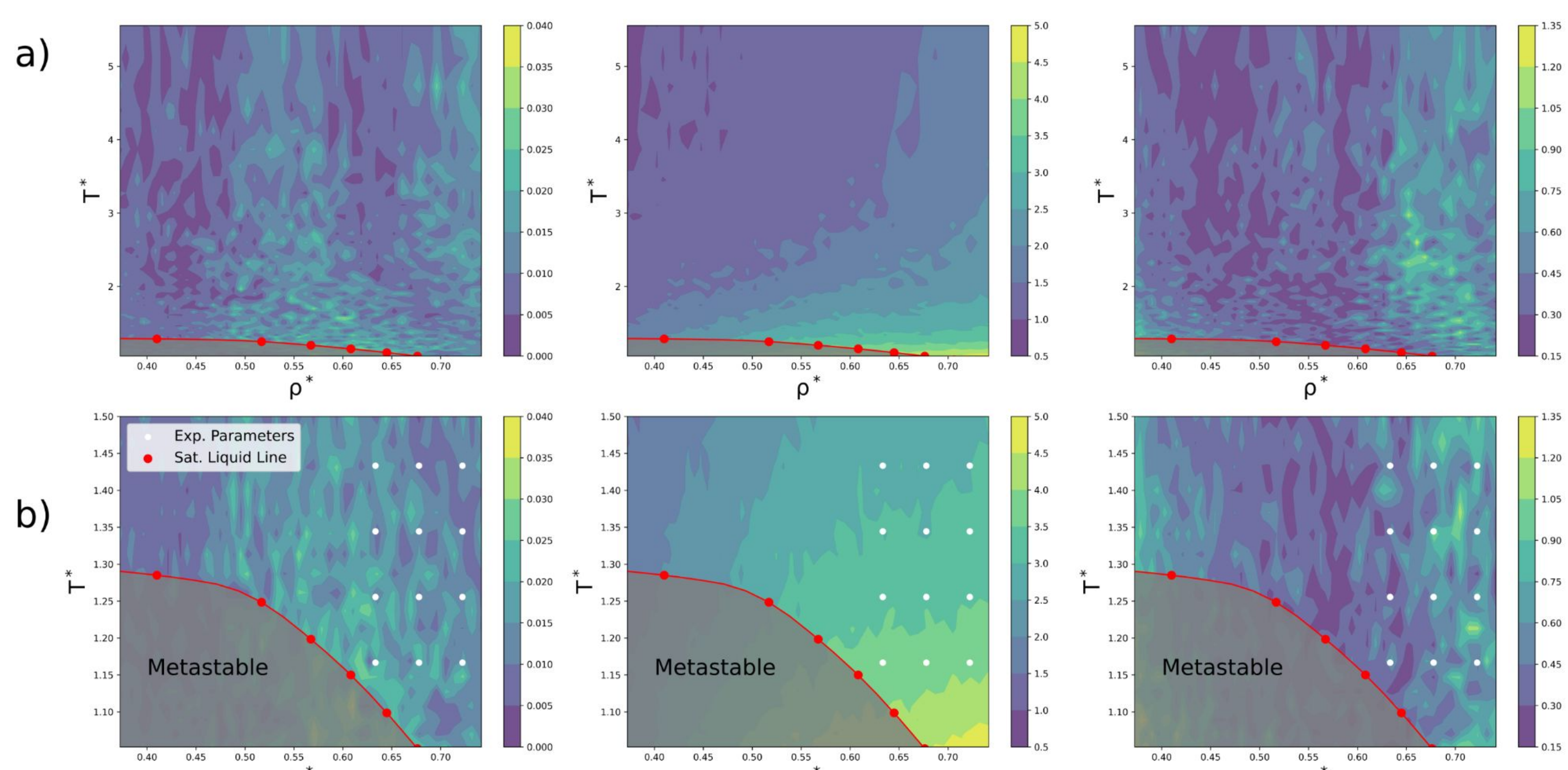


## QUANTIFYING STATE-DEPENDENT MANY-BODY CONTRIBUTIONS TO EFFECTIVE PAIR POTENTIALS

### SOPR meta-analysis of Krypton Radial Distribution Functions [5,6]



### Pair Potential Uncertainty and Sensitivity Varies Across the Phase Diagram



- Inferring atomic size from the WCA separation is consistent with the van der Waal radius and provide empirical evidence of a quantum drude oscillator type interaction
- SOPR potentials trained on experimental scattering data show a temperature and pressure dependence that manifests in different features of the potential.
- Sensitivity of the potential to the structure factor in simple liquids is more nuanced than previously reported by Weeks, Chandler and Anderson.

## CONCLUSIONS

1. Neutron scattering data is a viable target for force field training.
2. Structure inversion is a valuable tool beyond its typical use as a coarse graining method.
3. SOPR potentials can be analyzed to reveal fundamental insight into quantum mechanical behaviors.
4. SOPR potentials are state-dependent and may reveal fundamental insight into how many-body interactions change across the phase diagram.
5. It may be possible to identify supercritical phase transitions from scattering data and SOPR.

## REMAINING SCIENTIFIC QUESTIONS

1. Are there other quantum mechanical properties that can be inferred from SOPR potentials?
2. How can we rigorously quantify the many-body contribution of the SOPR potential?
3. Why do effective pair potentials have an apparent trend in equilibrium separation as a function of pressure and maximum attractive force as a function of temperature?
4. Do (n-6) Mie potential representations of the potentials exhibit the same trends and how confident are we that these potentials are correct?

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